Detailed Air-to-Water Heat Exchanger Model for a Multicomponent Solar Thermal System

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Abstract

A fin-and-tube heat exchanger model is presented in this paper. It uses empirical heat transfer and flow friction correlations identified in the literature. The model structure, its range of validity and accuracy are described in detail. Additionally, the model performance is compared with the producer design software GPC.

1. Introduction

Fin-and-tube heat exchangers are widely used in industry and residential air conditioning for heat transfer between a liquid and a gas, e.g. for water cooling, air cooling or heating. In the context of a research project in Bishkek (Kyrgyzstan) such a heat exchanger is applied in a multicomponent solar thermal system [1] to heat up cold water for a district heating net by using the enthalpy of the ambient air. The ambient air is also preheated with an unglazed transpired air collector before going through a heat exchanger.

In practice, the heat exchanger geometry is selected by designers relying on their personal experience and some recommendations1. If the heat capacity rate ratio of liquid to air is not defined by the application, a typical value (e.g. 2) is applied. This procedure works well for standard applications. But for non-standard applications with different boundary conditions (e.g. low electricity prices) as described above, there are more detailed investigations necessary. In order to optimize the heat exchanger configuration for this application, a detailed heat exchanger model was developed, which is presented in this paper.

2. Heat exchanger model

2.1. Model structure

There are in general two possible ways to describe the thermal behaviour of a heat exchanger considering its geometry. The first one is to (numerically) solve heat transfer equations using finite element or volume methods, e.g. in FLUENT. However, these models require much computing effort and are, therefore, not appropriate for an optimization of the whole heat exchanger configuration. Depending on the optimization algorithm and the complexity of a problem, several thousand calculations can be required. The second way is to use empirical heat transfer correlations

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2 For example, air flow velocity > 2 m/s for even distribution and < 3.5 m/s (otherwise pressure drop too high) [2]
gained from experimental investigations. Models based on empirical correlations require little computing effort, but usually have higher inaccuracies than those using finite element methods.

The model presented here (in the following simply referred to as “the model”) uses empirical heat transfer correlations. It is based on the detailed cooling coil model Type1223new\(^2\) of the ASHRAE\(^3\) HVAC Secondary Toolkit \([3, 4]\), which separately considers wet and dry parts of the heat exchanger surface. Thus, the model accounts possible condensation of vapour\(^4\) on parts or even all over the heat exchanger surface, if the surface temperature is below the air dew point temperature. In comparison to Type1223new, other heat transfer correlations have been implemented. Furthermore, pressure drops on both water and air sides are accounted and the model uses temperature-depending physical water and air properties, evaluated at the mean flow temperature, instead of constant values.

### 2.2. Airside heat transfer correlations

Type1223new uses airside heat transfer correlations of Elmahdy and Biggs \([5]\). The latter, however, is restricted to coils with circular or continuous plain fins and to the coil dimensions used in the experiment (9 samples with plain fins and 12 finned tube heat exchangers). In order to extend the validity range and the generality of the model, other correlations \([6-14]\) were identified in the literature, which had been generated from larger databases, sometimes including experimental data from previous reports. Unfortunately, different correlations cannot be directly compared in terms of heat transfer coefficients, because different data reduction methods\(^5\) were used, which makes it difficult to choose the “best” one.

Jacobi et al. \([14]\) reviewed many correlations and recommended the correlations of Wang et al. \([7]\) for dry surface and Wang et al. \([8]\) for wet surface for plain fin round-tube heat exchangers. Nevertheless, the more recent correlations of the same authors \([10, 11]\) have been implemented in the model, which include data of previous reports (in total 74 samples with dry surface and 31 samples with wet surface).

Plain fins are, however, not the current industry standard. Most fins have either waves or louvers for heat transfer augmentation. The heat exchanger used in the test plant in Bishkek has herringbone wavy fins. Therefore, the heat exchanger model was extended for herringbone/wavy fins by adding the correlation of Wang et al. \([12]\) for dry surface (61 samples in the data base) and Pirompugd et al. \([13]\) for wet surface (18 samples in the data base). If necessary, other correlations, e.g. for louvered fins, can be easily implemented as well. The switch between fin types and their respective correlations is done by a model parameter FinType (e.g. 0 – plain fins, 1 – wavy fins).

It has to be noted, that the same equations for liquid-side heat transfer, fin efficiency and \(\varepsilon\)-NTU relation have to be used to calculate the overall heat transfer rate as in the data reduction method for developing the airside correlation. All implemented airside heat transfer correlations of the model have the same data reduction method and its equations will be presented in the following paragraphs.

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\(^2\) Fortran source code of the model can be downloaded from the TRNSYS website.  
http://sel.me.wisc.edu/trnsys/trnlib/ASHRAE_secondary_toolkit/heat_and_mass_transf/1223NEW.for  
\(^3\) American Society of Heating, Refrigerating and Air-Conditioning Engineers  
\(^4\) Considering of condensation is of importance for the mentioned application in Bishkek, Kyrgyzstan as the dew point temperature often exceeds the water inlet temperature of 12°C.  
\(^5\) For example, the heat transfer coefficient calculated from (overall heat transfer) measurement data is lower if the heat exchanger is considered as a counter flow heat exchanger (the assumption valid for high number of tubes in air flow direction, e.g. \([6]\)) than that for a cross-counter flow heat exchanger (e.g. \([7-13]\)
2.3. Fin efficiency

Fin efficiency in Type1223new is calculated using an analytical solution for circular plain fins. This can, however, overestimate the fin efficiency for high performance heat transfer surfaces (fins with waves, slits and louveres) [16]. If the overall heat transfer coefficient is determined using the same fin efficiency calculation as in developing of a heat transfer correlation, no error will be generated. Therefore, the fin efficiency calculation according to Schmidt [15] (taken from [9]) was implemented in the model, which was used in developing of the implemented correlations.

2.4. Liquid-side correlation

For a liquid-side heat transfer correlation a simplified Gnielinski correlation Eq. (1) was implemented, which corresponds to the chosen airside heat transfer correlations [10-13].

\[
U_i = \left( \frac{k}{D_i} \right) \frac{\left( \frac{Re_{D_i} - 1000}{\Pr} \right) Pr(f_i/2)}{1 + 12.7 \sqrt{f_i/2} \left( \frac{Pr^{2/3} - 1}{Pr} \right)} \quad (1)
\]

with

\[
f_i = \left( 1.58 \ln \left( \frac{Re_{D_i}}{3.28} \right) \right)^2 \quad (2)
\]

This simplified correlation is only valid for turbulent and transitional flow. At small Re numbers (< 4000) heat transfer can be considerably over predicted with this correlation.

2.5. Heat transfer rate calculation

The overall heat transfer resistance is defined from the following relationship

\[
\frac{1}{UA} = \frac{1}{U_i A_i} + \frac{\ln \left( \frac{D_o}{D_i} \right)}{2 \pi L \cdot k_{tube}} + \frac{1}{U_i A_i \left( 1 - \frac{A_e}{A_o} (1 - \eta) \right)} \quad (3)
\]

The authors of the airside heat transfer correlations applied \( \varepsilon \)-NTU relationships for cross-counter flow heat exchangers from ESDU [17]. These relationships are used in the model instead of those in Type1223new for counter-flow heat exchanger. The number of heat transfer units, the capacity flow rate ratio \( C^* \) and the efficiency \( \varepsilon \) are defined as:

\[
NTU = \frac{UA}{C_{\min}} \quad (4), \quad C^* = \frac{C_{\min}}{C_{\max}} \quad (5), \quad \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\dot{Q}}{C_{\min} \left( T_{in,air} - T_{in,liq} \right)} \quad (6)
\]

Knowing NTU and \( C^* \), the efficiency can be calculated using a corresponding \( \varepsilon \)-NTU relationship. The total heat transfer rate is determined as:

\[
\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min} \left( T_{in,air} - T_{in,liq} \right) \quad (7)
\]

2.5. Pressure drop calculation

There is no pressure drop calculation in Type1223new, but this is important for optimizing the heat exchanger configuration. Therefore, a pressure drop calculation was added to the model.

The airside pressure drop is determined from corresponding empirical friction factor correlations [10-13]. The friction factor \( f \) includes a Fanning friction factor\(^6\) and parts of entrance and exit pressure drops, which are associated with irreversible free expansion that follows a sudden contraction or an abrupt expansion, compare with [18].

With the friction factor \( f \) the pressure drop is calculated as

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\(^6\) Fanning factor is defined as the ratio of wall shear stress to the flow kinetic energy per unit volume [18]
\[ \Delta p_{\text{air}} = \frac{G^2}{2 \rho_l^2} \left[ \frac{A_h}{A_c} \frac{\rho_l}{\rho_m} f + \left( 1 + \sigma^2 \right) \left( \frac{\rho_l}{\rho_g} - 1 \right) \right] \quad (8) \]

The liquid-side pressure drop in the model consists of a pressure drop in the passes (flow friction) and in the pass bends (form drag effects). The friction factor for tube passes is calculated according to the equation of Colebrook and White with typical roughness height for new copper tubes of 0.0013 mm [19]. The equation of Colebrook and White and an equation for friction factor of 180° bends can be found in almost any fundamental hydrodynamics book and will not be described here.

### 3. Validity range and accuracy of the model

The liquid-side heat transfer correlation is valid for high Re numbers (see section 2.4). For the airside performance correlations used in the model, the validity range is determined by the range of heat exchanger geometrical parameters and operating parameters in the data base. These are (here shown only for dry surface):

For plain fins [10]: \( \text{Re}_{\text{air}} = 300 \cdots 11000, \ N = 1\cdots6, \ \text{D}_o = 6.35\cdots12.7 \ \text{mm}, \ F_p = 1.19\cdots8.7 \ \text{mm}, \ P_t = 17.7\cdots31.75 \ \text{mm}, \ P_l = 12.4\cdots27.5 \ \text{mm}. \)

For herringbone wavy fins [12]: \( \text{Re}_{\text{air}} = \) not explicitly mentioned, but should be in a similar range as in the correlation above, \( N = 1\cdots6, \ \text{D}_c = 7.66\cdots16.85 \ \text{mm}, \ P_d = 0.3\cdots1.8 \ \text{mm}, \ F_p = 1.21\cdots6.43 \ \text{mm}, \ P_t = 21\cdots38 \ \text{mm}, \ P_l = 12\cdots33 \ \text{mm}. \)

Although a corrugation angle is one of the parameters in the correlation, its range is not given in [12]. Additionally, wave height \( P_d \) or wave length \( x_f \) are not considered in the correlation, although they have a considerable influence on heat transfer and friction coefficients at \( \text{Re} > \text{Re}_{\text{critical}} \) [20]. Furthermore, the influence of the corrugation angle in the heat transfer correlation is inconsistent. For some geometrical parameter combinations heat transfer coefficient expressed in terms of Colburn factor increases with increasing corrugation angle as expected or stated by many investigations (cf. literature review in [20]), Fig. 1a. For other parameter combinations it does not, even having sometimes a clear optimum, Fig. 1b. Whereas the friction factor increases with increasing fin corrugation angle as expected, Fig. 2. Additionally, for some fin corrugation angle values the Colburn factor \( j \) and the friction factor \( f \) for wavy fins are smaller than those for plain fins (calculated according to [10]), Fig. 1 and 2. These effects are obviously caused by a complex structure of the correlation. Unfortunately, no other (“better”) correlations for wavy fin heat exchangers were identified in the literature, nor did Jacobi et al. [14] find any.

### Table 1: Geometrical dimensions of the heat exchangers referred to in this paper.

<table>
<thead>
<tr>
<th>No.</th>
<th>N</th>
<th>( P_t )</th>
<th>( P_l )</th>
<th>( F_p )</th>
<th>( \delta_f )</th>
<th>( \text{D}_c )</th>
<th>( \theta )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>HX 1</td>
<td>6</td>
<td>50</td>
<td>25</td>
<td>3</td>
<td>0.25</td>
<td>12.5</td>
<td>15</td>
<td>Güntner F</td>
</tr>
<tr>
<td>HX 2</td>
<td>6</td>
<td>25</td>
<td>22</td>
<td>3</td>
<td>0.15</td>
<td>10.1</td>
<td>15</td>
<td>Güntner H</td>
</tr>
<tr>
<td>HX 3</td>
<td>2</td>
<td>25.4</td>
<td>19.05</td>
<td>3.09</td>
<td>0.12</td>
<td>8.62</td>
<td>NA</td>
<td>No. 44 in [12]</td>
</tr>
</tbody>
</table>
Fig. 1. Influence of the fin corrugation angle on a Colburn factor $j$ correlation [12] for two heat exchanger configurations: (a) left HX 2, (b) right HX 3, see Table 1. For comparison, values of the Colburn factor for plain fins [10] are also shown.

Fig. 2. Influence of the fin corrugation angle on a friction factor $f$ correlation [12] for two heat exchanger configurations: (a) left HX 2, (b) right HX 3, see Table 1.

All chosen airside correlations have similar accuracies. They describe about 90% of experimental heat transfer data (Colburn factor) and approx. 85% of friction factors within 15% error.

The flow friction usually contributes more than 90% of the airside pressure drop in a heat exchanger [18], so that the model accuracy regarding the airside pressure drop calculation is in a similar range as the accuracy of the airside friction factor correlation. An accuracy of equations for friction factor in tubes is not given in [19]. Thus, the model accuracy regarding the pressure drop calculation on liquid-side is unknown.

The heat transfer rate is not directly proportional to the airside heat transfer coefficient. Therefore, a differential sensitivity analysis was carried out in order to estimate the influence of the uncertainty of the airside heat transfer coefficient on the heat transfer rate. The results for the heat exchanger geometry HX1 (see Table 1) and two different ratios of UA values on liquid and air sides are shown in Fig. 3, where the airside heat transfer coefficient was varied up to 20%. The $U_{Ao}$ value on the air side changes less than the $U_o$ value, because the fin efficiency is negatively correlated with $U_o$. The overall UA value is less sensitive than $U_{Ao}$, but its sensitivity depends on the ratio of $UA_i$ and $UA_o$. For both presented typical $UA_i$ and $UA_o$ ratios, the change in the heat transfer rate remains less than 5%, when the airside heat transfer coefficient varies within ±20%. Thus, the model accuracy regarding heat transfer calculation is within 5%, when the uncertainty of the airside heat transfer coefficient is less than 20% (at least for the investigated geometry).
4. Comparison with a producer design software

The model performance with 2 different airside correlations (Wang et al. [12] for wavy fins and Wang et al. [10] for plain fins) was compared with Type1223new and the producer design software Güntner Product Calculator (GPC) of the company Güntner GmbH. Two staggered tube layouts available in GPC (HX 1 and HX 2, heat exchanger length of 1.25 m and height 1 m, 10 passes) with wavy fins (corrugation angle = 15°) were considered. For both geometries the heat transfer rate, calculated by the model with the correlation for wavy fins, is about 10% for HX 1 and less than 5% for HX 2 lower than that of the GPC, (Fig. 4). It has to be noted, that transverse tube pitch \( P_t \) in HX 1 is out of the validity range of the correlation for wavy fins, extrapolation of the correlation is in general not recommended. The heat transfer rate, calculated by the model for plain fins, is lower than the GPC heat transfer rate too (Fig. 4), which is plausible. However, HX 1 is also out of the validity range of the plain fin correlation, the deviation of this correlation to the GPC for HX 1 is even smaller than that of the wavy fin correlation, probably because of the less complex structure. Unlike these correlations, Type1223new with the Elmahdy and Biggs [5] correlation for plain fins gives higher heat transfer rate values than the GPC. It is, however, unfeasible that plain fins have higher heat transfer coefficient than wavy fins (compare with [20]).

The airside pressure drop, determined with the correlations for plain and wavy fins, is significantly lower than that, calculated by the GPC, Fig. 5. Whereas liquid-side pressure drop calculation slightly overestimate the pressure drop calculated by the GPC and is in agreement with the GPC when calculated as smooth tubes.

Fig. 4 Heat transfer rate calculated by the model (as plain and wavy fins), Type1223new and GPC for different air flow rates and two geometries (left for HX 1, right for HX 2).
5. Conclusion

A model for fin-and-tube heat exchangers, which is based on empirical heat transfer and flow friction correlations, is presented here. The selected correlations are developed with larger data base and have complete description of the reduction method than the one used in Type1223new. In general one needs to be careful with empirical correlations, especially with complex ones, and one needs to prove simulation results. It is not recommended to extrapolate the correlations for configuration outside of the validity range. If configuration outside of the validity shall be simulated (e.g. for optimization of heat exchanger configuration) it appears to be sensible to use less complex correlations, e.g. Wang et al. [10] instead of Wang et al. [12].

Acknowledgements

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Nomenclature

\begin{align*}
A_f & \quad \text{m}^2 \quad \text{minimum flow area} & \dot{Q} & \quad \text{W} \quad \text{heat transfer rate} \\
A_{\text{face}} & \quad \text{m}^2 \quad \text{frontal area} & \text{Re} & \quad \text{Reynolds number} \\
A_i & \quad \text{m}^2 \quad \text{tube inside surface area} & T & \quad ^\circ\text{C} \quad \text{temperature} \\
A_o & \quad \text{m}^2 \quad \text{total airside surface area} & U & \quad \text{W/m}^2\text{K} \quad \text{heat transfer coefficient} \\
C & \quad \text{W/K} \quad \text{heat capacity rate} & \delta_f & \quad \text{m} \quad \text{fin thickness} \\
D_c & \quad \text{m} \quad \text{collar diameter} & \varepsilon & \quad \text{heat exchanger effectiveness} \\
D_l & \quad \text{m} \quad \text{tube inner diameter} & \eta & \quad \text{fin efficiency} \\
D_o & \quad \text{m} \quad \text{tube outer diameter} & \theta & \quad ^\circ \quad \text{fin corrugation angle} \\
f & \quad \text{friction factor} & \rho & \quad \text{kg/m}^3 \quad \text{density} \\
G & \quad \text{kg/m}^2\text{s} \quad \text{air mass flux based on minimum flow area} & \sigma & \quad \text{ratio of minimum flow area to face area} \\
J & \quad =\text{Nu}/(\text{RePr}^{1/3}) & \text{Subscripts} & \; \\
k & \quad \text{W/mK} \quad \text{thermal conductivity} & 1 & \quad \text{airside inlet} \\
N & \quad \text{number of tube rows} & 2 & \quad \text{airside outlet} \\
P_l & \quad \text{m} \quad \text{longitudinal tube pitch} & i & \quad \text{tube inner side} \\
P_t & \quad \text{m} \quad \text{transverse tube pitch} & m & \quad \text{mean} \\
Pr & \quad \text{Prandtl number} & \text{min} & \quad \text{minimum value}
\end{align*}
\( r_m \)  
\( \text{tube inner radius} \)  
\( \max \)  
\( \dot{Q}_{\max} \)  
\( \text{maximum possible heat transfer rate} \)  
\( o \)  
\( \text{airside} \)

**References**


